1 Week 6 HOGU: Sections 3.1 - 3.4

Problem 1. A small convenience store manufactures two different types of shampoo - Pantene Shampoo and Tresemme shampoo. The profit of production for each bottle of shampoo is \$6 and \$9 respectively. The stores calculates that no more than 5 bottles of shampoo (whether from Pantene or Tresemme) are bought each week.

The cost to the company from stocking Pantene shampoo is \$2 per bottle, while the cost to the company from stocking Tresemme shampoo is \$3 per bottle. The company does not plan on making spending more than \$12 each week on stocking these shampoo products.

What is the maximum profit the company can make from selling Pantene and Tresemme shampoo in a week?

(a) Set up the variables of this equation. What is the objective function? Are you maximizing or minimizing that function?

(b) Set up the constraints for this linear programming problem.

Problem 2. *Set up, but do not solve,* the linear programming problem below.

A factory produces two types of gas, regular and premium. Each gas requires the use of two operations: distillation and treatment. Regular gas requires 1 hour of distillation and 2 hours of treatment per batch, while premium gas requires 1 hour of treatment and 2 hours of distillation per batch. The factory has only 24 hours they can devote to distillation and 36 hours they can devote to treatment. They also can only make at most twice as much premium gasoline as regular gasoline.

If the factory makes \$20 of profit for each batch of regular gasoline and \$30 of profit for each batch of premium gasoline, how many of each should be manufactured to maximize profit?

Variables:

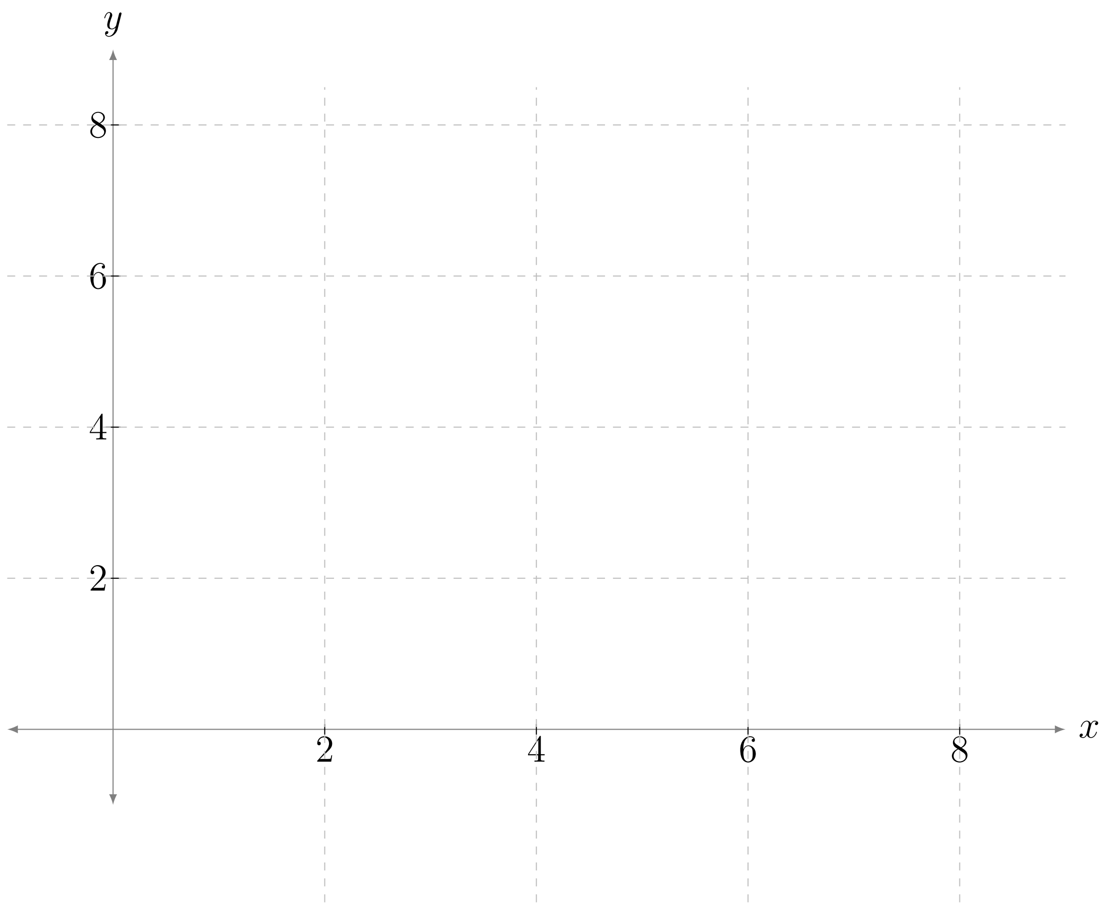
Objective: Maximize/Minimize (circle one) ______________________

Subject to:

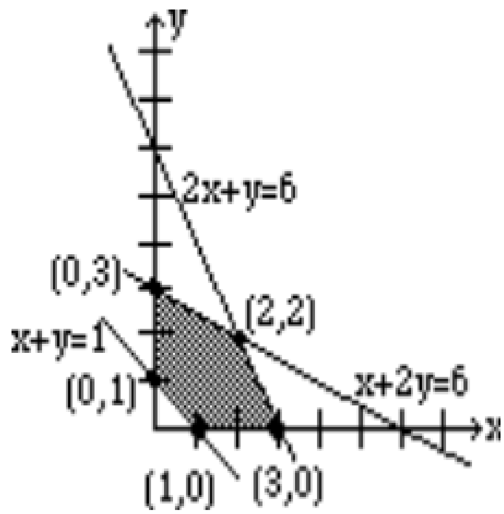
Problem 3. Graph the boundary lines for the following inequalities:

$$\begin{aligned} 0 \le x &\le 5 \\ 0 \le y &\le 5 \\ x + y &\le 8 \\ x &\ge \frac{1}{3}y \end{aligned}$$

Use a shading technique to find the solution set for this set of inequalities. Mark it with an "**S**".



Problem 4. Below is the feasible region for a linear programming problem:



We know our objective function for this linear programming problem is $P = 10x + 15y$, but we do not know if we want to maximize or minimize it.

(a) Does a maximum exist for this linear programming problem? Why? How about a minimum?

(b) Find the maximum value and the minimum value of this objective function for this linear programming problem. (If one value doesn't exist, just write DNE.)

Maximum: ____________________

Minimum: ____________________

Problem 5. John is solving a linear programming problem with an objective function of $P = 5x + 10y$. He notices that his feasible region is unbounded in Quadrant I and that the corner points of this region are $(2, 0)$, $(1.5, 0.25)$, and $(0, 0.5)$. What is the maximum value of his objective function in this region, and what point(s) yields this value?

(A) $P = 10$ at $(2, 0)$ and $(1.5, 0.25)$

(B) $P = 10$ at any point on the line segment connecting $(2, 0)$ and $(0, 0.25)$

(C) $P = 5$ at $(0, 0.5)$

(D) $P = 20$ at $(4, 0)$

(E) There is no maximum value in this region.

Problem 6. (a) Write a simplex tableau for the following linear programming problem: **Maximize**: $P = x + 9y + 4z$ **Subject to**:

$$\begin{aligned} \frac{2}{7}x + \frac{1}{5}y + \frac{3}{4}z &\leq 630 \\ \frac{1}{4}x + \frac{1}{8}y + \frac{1}{2}z &\leq 250 \\ x - y + 2z &\leq 480 \end{aligned}$$

(b) What is the first pivot entry for this tableau? Give the row number and column number for this entry.

Problem 7. For each simplex tableau, determine whether it is in its final form. If it is, find the optimal solution. If it is not, state the next pivot entry.

(a)

x	y	s_1	s_2	s_3	s_4	P	const
2	3	1	0	0	0	0	6
−3	2	0	1	0	0	0	3
0	2	0	0	1	0	0	5
2	1	0	0	0	1	0	4
−4	−3	0	0	0	0	1	0

(b)

x	y	s_1	s_2	s_3	P	const
2	8	1	0	0	0	56
3	2	0	1	0	0	24
−1	4	0	0	1	0	0
−10	−5	0	0	0	1	0

(c)
$$\begin{array}{ccccc|c} x & y & s_1 & s_2 & P & \text{const} \\ 1 & 0 & \frac{3}{4} & -\frac{1}{4} & 0 & 3 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & 0 & 4 \\ \hline 0 & 0 & \frac{3}{2} & \frac{1}{2} & 1 & 24 \end{array}$$

(d)
$$\begin{array}{ccccc|c} t & f & s_1 & s_2 & P & \text{const} \\ 0 & 9 & 1 & -3 & 0 & 486 \\ 1 & \frac{3}{2} & 0 & \frac{1}{2} & 0 & 99 \\ \hline 0 & 10 & 0 & 30 & 1 & 5940 \end{array}$$

Problem 8. A Math Learning Center tutor rolls two six-sided dice, one green and one blue, noting the side facing up when they land.

Let E be the event "the sum of the two dice is even". Let F be the event "a 4 is rolled on the blue die". Let G be the event "the green die shows a number greater than 7".

(a) How many outcomes are there in G^C?

(b) Verbally describe the outcomes in the event $E \cap F$.

(c) List the outcomes in $E \cap F$.

Problem 9. Let $S = \{\triangle, \square, \blacktriangleright, a, b, c, 1, 2, 3\}$ be the universal set. Let $A = \{\square, a, b, 3\}$ and let $B = \{2, \blacktriangleright, a\}$.

(a) How many possible events are there in S?

(b) Are A and B mutually exclusive? Why or why not?

(c) Give the set $(A \cup B)^C$.

(d) Give the set $A \cup B^C$.